

CONFINEMENT AND LATTICE GAUGE
THEORY

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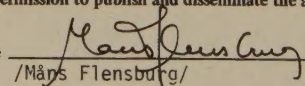
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Abstract In the first paper of this thesis we explore the phenomenological consequences of an improved MIT bag model. Smaller hadronic radii and larger bag constant results from a CMS correction. The new values are found to be consistent with various dimensional parameters in hadron physics. We also calculate the mixed qqg spectrum. Paper II and III deals with an analysis of all existing high statistics lattice Monte Carlo data on the static quark potential for $SU(3)_4$, in terms of an effective string theory. Good agreement is found. In paper IV we perform a full $SU(2)_2$ lattice Monte Carlo calculation at finite temperature. We determine the deconfinement temperature in the scaling region and compare it with the predictions of the scalar string. The two final papers investigate the possibility to use the complex Langevin equation in generating lattice configurations, including charges, for $U(1)$, $SU(2)$ and $SU(3)$. The power of the method is illustrated for $U(1)$ in two dimensions, while the method fails in the non-abelian cases.			
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This thesis is based on the following publications

I Applications of an Improved Bag Model

Z. Phys. C22, 293 1984

(with C. Peterson and L. Sköld)

II Strings and SU(3) Lattice Gauge Theory

Phys. Lett. 153B, 412 1985

(with C. Peterson)

III String Model Potentials and Lattice Gauge Theories

Lund Preprint LU TP 85-14

(submitted to Nucl. Phys. B)

(with C. Peterson)

IV Measurement of T_c in the scaling region of 2+1 dimensional SU(2)
Lattice Gauge Theory

Lund Preprint LU TP 86-7

(submitted to Phys. Lett. B)

(with A. Irbäck)

V Monte Carlo Simulations of Configurations with Static Charges

Phys. Lett. 159B, 335 1985

(with J. Ambjorn and C. Peterson)

VI The Complex Langevin Equation and Monte Carlo Simulations of
Actions with Static Charges

Lund Preprint LU TP 86-6

(submitted to Nucl. Phys. B)

(with J. Ambjorn and C. Peterson)

Introduction

Simple in its concept, though rich in its structure.

QCD, Quantum Chromo Dynamics, has emerged in the last decade as the theory of the strong interaction phenomena. The success of effective extended models like the "bag" model [1], and the "string" model in spectroscopy and dynamical quark fragmentation respectively, demonstrate the richness and complexity that seems to be inherent in QCD if it is to be the underlying local quantum field theory. QCD as a non-abelian gauge theory in color space has been singled out as the sole candidate, for a strong interaction theory.

The effective models each hints at one of two outstanding features of QCD. First; confinement. Unlike usual quantum numbers, color charges do not lead to any directly observable spectroscopy. Only singlet states under the color symmetry seem to be realized. Second; asymptotic freedom, resulting in the approach of a non-interacting theory at small scales. This is the pointlike high energy regime where perturbation theory is perfectly valid and the resulting predictions can be compared with experiments. We may interpret these features in the notion of a scale dependent "running" coupling constant which decreases as the momentum-space cutoff increases.

The establishment of asymptotic freedom in QCD with the local non-abelian gauge symmetry of $SU(3)$, together with the numerical demonstration of confinement at long range, have had an enormous impact on the development of elementary particle theory.

Yet, QCD has not been solved in the confinement region. The topic of this work is the development of methods to allow the study of this regime, either numerically or by means of effective theories. The first part of the thesis is devoted to a study of a bag model with the small

scale dynamics of QCD incorporated in a perturbative fashion. The latter and major part deals with and uses one of the few, and perhaps the only really powerful, method to study the behaviour of QCD in its nonperturbative region of confinement; gauge theory on a lattice.

Lattice gauge theory

Standard regularization prescriptions are based on the removal of divergences diagram by diagram in a perturbative expansion of some specific quantity. To have a theory well defined outside the perturbative region a fixed cutoff, with no reference to perturbation theory, should be used instead.

K.G. Wilson realized that this could be gauge invariantly achieved by addressing the fields of the symmetry group to the links of a space-time lattice [2]. The action is defined in terms of the sum over link variables multiplied along a small square loop, a plaquette. The lattice spacing (a) will constitute our natural momentum cutoff. It is the existence of a critical point in such a theory that will allow for the definition of a local relativistic and scale invariant field theory in the close vicinity of this point at zero coupling. Remember that critical points are characterized by a diverging correlation length telling us that the specific small scale features of the theory become irrelevant.

Though the original motivation for the development of the theory was to allow for a strong coupling expansion that could be extended to small couplings, it was realized after some time that the continuation of the action to Euclidean space transformed the problem to one of statistical mechanics with a large but finite number of degrees of freedom, suitable for computer handling [3]. A simultaneous explosion of computer capacity has made the outstanding achievements in this area during the last few years possible.

In the following a short summary of the papers in this thesis will be presented.

Paper I

We explore some of the phenomenological consequences of a MIT bag model improved by corrections originating from the finite extension of the confining volume of hadrons.

The MIT bag model has since its proposal in 1974 proved to be a very successful way to consistently parametrize many aspects of hadron physics and especially hadron spectroscopy. Its main virtues lie in its simple and intuitive structure; a static surface, defining a perturbative inner region of a hadron, is kept by the pressure $B^{1/4}$ exerted on this surface by the lower vacuum energy density outside. B will thereby be related to QCD vacuum via parameters (such as the gluon condensate). Of course the QCD coupling strength α_s will also enter as a parameter, together with the strange quark mass m_s and the quark self energy e_q , the latter in principle calculable within the model. The Casimir energy is usually incorporated in e_q . The spectrum of the model is obtained by ordinary perturbation theory in terms of the free gluon ground states of the spherical cavity of radius R_0 . Finally R_0 is fixed by minimizing the energy and the remaining four parameters are determined by a fit to the low-lying hadron masses.

The modified bag model we explore has two more features which induce large effects upon the results. First, assuming the quarks to carry the total momentum $\langle P^2 \rangle$, we take some account for momentum conservation by minimizing $m(P)$ rather than E itself since E is the static bag Hamiltonian eigenvalue. The small masses will thereby decrease. Second, the asymptotic freedom is incorporated by using the running coupling constant mentioned above.

It has been shown that the modifications result in a significantly smaller hadronic radius R_0 than in the conventional model. Using the new values of $B^{1/4}$ and R_0 we explore the effects upon phenomenologically observable dimensional quantities such as the gluon condensate, the Hagedorn temperature T_H and the deconfinement temperature T_C . Whereas T_H enters in a thermodynamic description of the



transition from the bag to the quark continuum, T_c is defined here to be the temperature where the bag pressure $B^{1/4}$ is balanced by the gluon kinetic pressure alone. We find that the new parameters bring the gluon condensate value into agreement with the value obtained by use of charmonium sum rules, while the old smaller value is outruled by a factor of almost 2. Furthermore, the string tension is evaluated with the new parameters in a fluxtube model of the $c\bar{c}$ spectrum. Finally the smaller R_0 will affect many other areas of hadron phenomenology including geometrical crosssections and p_t spectra. In particular we examine the effect on weak decays where overlap integrals behave as $\sim R_0^{-3}$ in the bag model, and find nice agreement with experimental data.

The last part of the paper is devoted to a calculation of the hermafrodite spectra, i.e. the spectra of bound $q\bar{q}g$ states. The outcome is in good agreement with earlier works in spite of the very different parameters used. We find this quite reassuring. The exotic 1^{-+} state comes out with mass around 1400 MeV.

Paper II

The chromo-electric flux between color charges in the nonperturbative region of QCD is supposed to be trapped inside a thin vortex-like structure. This means that it should be possible to describe the long-range behaviour of QCD by an effective action build from the conjecture of a string as the fundamental entity. Such theories have been extensively studied and are in general hampered by divergences in the ultraviolet limit. We are however only interested in describing the behaviour at large distances.

It has been known for some time that a very general class of such conformally invariant string actions will exhibit quantum fluctuations in the transverse degrees of freedom. The quark anti-quark static potential at distance R in d space-time dimensions will thereby be modified by a, to lowest order universal, "roughening" contribution $\sim (d-2)/R$ [3].

Three-dimensional lattice Monte Carlo data have previously been confronted with the effective string theory. Excellent agreement was found. We have carefully adopted the analysis to the more interesting case of $SU(3)$ in four dimensions. Not only do we have to deal with lower statistics available due to the much larger number of degrees of freedom, but also does the perturbative part of the potential mimic the functional form of the roughening term and extend to larger distances than in $SU(2)_3$.

Two features in the fitting procedure should be stressed. First, the use of finite temporal size Wilson loops in extracting the potential introduces T dependent terms in the analog of the $(d-2)/R$ term. Second, the string description breaks down at some small distance R_c of the order of .5 correlation length. Below R_c the perturbative potential is dominant and these data point should be excluded in the fit.

In spite of these subtleties we succeed in fitting several sets of data of sufficient statistic to the string form, and we obtain values of the string parameters in accord with the model for all the sets. We have thus determined a new roughening corrected string tension s .

Paper III

We pursue the investigation in paper II to include further recent Monte Carlo data. While universal to lowest order, different effective scalar string actions differ at higher order. Systematic deviations when fitting to the string form in paper II, indicated that the data might allow for a direct determination of which effective action is the preferred one by QCD. This study is developed here. Finally we investigate and compile the results of string roughening corrected ratios $T_c/\sqrt{\sigma}$, where T_c is the deconfinement temperature, and compare with the prediction from strings.

Paper IV

Here we perform a full $SU(2)_3$ lattice calculation in order to determine the deconfinement temperature. We address the question of whether or not the effective string theory can be used when describing the approach to the deconfining phase transition in 2+1 dimensional $SU(2)$.

On a lattice with periodic temporal boundary conditions, the temperature is defined by the inverse temporal length of the lattice aN_t . In a picture very similar to the one leading to an effective string for distances beyond some critical R_c , one also obtains a breakdown of the effective string theory for temporal extension of the world sheet of the string below some critical periodicity N_t i.e. for temperatures beyond some critical T_c . There are some very appealing features of the string theory close to its breakdown. One is that the string tension goes to zero in a continuous way, just as the $SU(2)$ lattice entity does. Another is that the breakdown may be viewed as a delocalization, similar to the roughening, of a string of idealized small extension [4]. Eventually the tube will fill all space and its interior will describe the free gluon gas encountered for temperatures above T_c . It must be remembered however that we are here studying the string theory closer to its limits than was the case in the determination of $\sigma(R)$ in paper II.

Any serious attempt to determine T_c/σ should evaluate the quantity in a region where scaling in β can be established. For $SU(2)_3$ the σ extracted according to the recipe in paper II scales for $b > 5.0$. Since T_c is of more local origin, i.e. more sensitive to the lattice cutoff, scaling is expected to set in at still higher β . We also point out the importance of using the σ corrected for roughening when evaluating T_c/σ .

Using different quantities such as Wilson line expectation value and spatial correlations as well as local energy density, we determine T_c on fairly large lattices ranging up to 6×60^2 and β up to 9. We find scaling only for $b > 7.0$ and using a roughening corrected σ we get $T_c/\sigma = .94 \pm .03$, now in perfect agreement with the scalar string

prediction $\sqrt{3/\pi}$.

Paper V.

In conventional lattice Monte Carlo methods, vacuum configurations are generated according to a Boltzmann distribution. When measuring other than vacuum quantities the major part of this ensemble is completely irrelevant to the averages we are interested in. Long range objects like strings therefore have to be evaluated from exponentially damped projections. Could we generate configurations including charges the gain would be tremendous. Areas of physics completely inaccessible today could then easily be investigated.

In an Euclidean formulation source terms in the action will necessarily come out imaginary, rendering the Boltzmann weight $\exp(-S)$ non-positive. The probability interpretation of the weight is then not allowed and most conventional Monte Carlo methods fail, or can at least not be applied to non-abelian gauge groups. However, by using the stochastic Langevin equation, the drift $-\Delta S$ will drive the fields complex. Still, the underlying dynamics governed by a Fokker-Plank equation will be relevant. Though not proved it might be possible that the Langevin equation generates the relevant complex distributions, from which real quantities can be extracted [5].

We test the algorithm for the soluble case of compact $U(1)$ in one and two dimensions. In the latter case we impose bag-boundary conditions on a 20^2 lattice and measure the string tension for large separations with high accuracy. The method is thus shown to give correct values where conventional methods would yield Wilson loop expectation values $\sim 10^{-12}$. Here the potential power of the algorithm is clearly demonstrated.

Paper VI

Encouraged by the success of the complex Langevin equation for compact $U(1)$ lattice gauge theory in two dimensions we pursue the study to the

higher dimensional $U(1)$, where we do not have the factorization to the simple case of 1-dimensional $U(1)$. We implement a floating local steplength with multihit in order to cope with run-away solutions. This allows the results in paper V to be extended over more than one order of magnitude in β ! We also test the non-abelian cases of $SU(2)$ and $SU(3)$. In the $U(1)$ case, the energy densities E^2 , in presence of charges, have been explicitly calculated using a dual algorithm. The results are however neither in agreement with results from measurements of the string tension from conventional Wilson loops, nor do they seem to fit the continuum formula derived by Polyakov for the Wilson action. This makes $U(1)_3$ interesting in its own right.

When applying the complex Langevin equation to the 2+1 dimensional $U(1)$ we find that the algorithm converges to a dipole field in the weak coupling phase. We feel assured that the method works here. However, when we proceed to the strong coupling regime where a string of constant energy density should be clearly visible, we have to conclude that the technique fails.

For the non-abelian case the action will resemble an alternative abelian $U(1)$ action incorporating a charge term of the type $\log(\cos\theta)$. Here the algorithm fails at strong coupling. We give a reason for the failure in terms of the flow diagram of the drift ΔS . This argument does not directly apply to the 1-dimensional $SU(2)$ and $SU(3)$ case. Still, we find the same pattern of performance here.

Whereas strong coupling constitutes a nice testing ground for the algorithm, the relevant physical results are of course obtained in the weak coupling continuum region, where the method so far works well. Also, since 2-dimensional $SU(N)$ do not trivially factorize to the 1-dimensional case, the conclusive test must be performed in two dimensions. Surprisingly we find no signal of any rise of E^2 at all, for any coupling.

Still, the complex Langevin algorithm have recently been successfully applied to the non-linear sigma model. The outcome of this investigation is therefore that care should be exercised when applying the technique to new areas of physics. In each case the method must

first be compared with results obtained by standard methods.

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